Feb 14.

$$\frac{\partial g}{\partial t} = -2\, Ric(g)$$

solution space is not finite dim. (invariant under diffeo) $\longrightarrow$ not a parabolic question.

2nd Bianchi id

$$g^{ij} \nabla_i R_{jk} = \frac{1}{2} \nabla_k \underbrace{R}_{g^{ij} R_{ij}}.$$

using $\nabla g = 0$

$$\Leftrightarrow \quad g^{ij}\left(\nabla_i R_{jk} - \tfrac{1}{2}\nabla_k R_{ij}\right) = 0$$

Define an operator

$$L(g_{ij})(T_{kl}) = g^{ij}\left(\nabla_i T_{jk} - \tfrac{1}{2}\nabla_k T_{ij}\right)$$

$$L : \Gamma(\Lambda^2(TM)) \longrightarrow \Gamma(Hom(\otimes^2 T^*M, T^*M))$$

the Bianchi operator $\qquad$ integrability cond.

Let $E = -2Ric$, then

$$\underbrace{L(g)}_{\deg 1} \underbrace{DE(g)}_{2} \tilde{g} + \underbrace{DL(g)\{E(g), \tilde{g}\}}_{\deg < 3} = 0$$

$$\sigma(L(g))(\xi) \circ \sigma(DE(g))(\xi) = 0$$

$$\Rightarrow \operatorname{im}(\sigma(DE(g))(\xi)) \subseteq \ker(\sigma(L(g))(\xi))$$

So we can consider the action

$$\sigma(DE(g))(\xi) : \ker(L(g)(\xi)) \circlearrowright \qquad \underline{\text{Thm}}$$

$$f \longmapsto |\xi|^2 f$$

Hamilton

assume: $E : C^\infty(X, F) \xrightarrow[\text{deg } 2]{\text{nonlinear}} C^\infty(X, F)$

F: vector bundle on X

$L : C^\infty(X, F) \xrightarrow{\deg 1} C^\infty(X, \mathrm{Hom}(F, G))$

G: vector bundle on X.

$$L(f)(E(f)) = \underline{Q(f)}.$$

deg at most 1 in f.

linearization

$$L(f)\, DE(f)\, \tilde{f} + DL(f)\{E(f), \tilde{f}\} = DQ(f)\tilde{f}$$

linearization of a degree k operator is deg k.

Hamilton Thm

Given $\dfrac{\partial f}{\partial t} = E(f)$ with the integrability condition $L(f)$

(A) $L(f)(E(f)) = Q(f)$ has deg at most 1.

(B) $\sigma(DE(f))(\xi) : \ker \sigma(L(f))(\xi) \circlearrowright$ has all the eigenvalues with positive real parts.

Then the IVP $f = f_0$ at $t=0$ has a unique smooth solution for a short time $0 \le t \le \varepsilon$.

Nash-Moser inverse function thm.

Let F, G be tame Frechet space, $U \subseteq F$ open subset. Suppose $P : U \subseteq F \longrightarrow G$ a smooth tame map, $\forall f \in U$.

$$DP_f : F \to G \quad \text{is invertible}$$

the family $Q: U \times G \longrightarrow F$ is tame smooth.

$$(f, \tilde{g}) \longmapsto (DP_f)^{-1} \tilde{g}$$

Then P is locally invertible, and the local inverse of P is smooth tame.